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TWO-DIMENSIONAL PROBLEMS OF BEAM FORMING UNDER CONDITIONS OF CREEP

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Direct and inverse problems of forming of long-length profiles with double curvature and a given angle of twisting under conditions of creep are considered. A finite-difference scheme for the numerical solution is proposed. Examples of solving problems with different types of external actions for a profile with a rectangular cross section are given. Experimental and numerical data are compared for twisting of beams with square and circular cross sections in the regime of creep at temperatures of 725 *and* 740°C *for* St. 45 *steel.*

Introduction. Long-length double-curvature profiles of variable cross section (such as rectangles, T-beams, flange beams, etc.) made of alloys hardly deformed at normal temperature are used as reinforcing elements of sheathing of aircraft, ships, and other machines. These profile have the following typical dimensions: lengths of several meters, wall height of 0.02–0.2 m, and wall thickness of several millimeters. One possible method of forming these profiles from straight-line beams is consecutive deformation of their sections. Each of the sections formed is subjected to temperature and force actions in a heat chamber. Some part of the profile is deformed under conditions of creep, and the other part of the profile located outside the chamber is not loaded. Under these conditions, we may assume in the first approximation that constant curvatures and a constant running angle of twisting along the profile are specified for the deformed section. Then the independent variables are two spatial coordinates in the plane of this section and the time. Hereinafter, the angle of twisting is understood as the angle of turning of the profile section per unit length.

Owing to the large (by an order of magnitude) difference in the size of the cross section and the length of the deformed section, the special feature of forming of long-length profiles is significant elastic recovery. Therefore, the specified residual curvatures and the angle of twisting can differ significantly from the curvatures and angle of twisting before unloading. The forming process is described by the following inverse problem: to find appropriate force and kinematic parameters of forming for obtaining required residual curvatures and angle of twisting after unloading and elastic recovery.

Similar inverse problems arise in forming of smooth and reinforced monolithic panels and arcs [1]. Problems of mathematical well-posedness of some inverse problems for smooth plates were considered in [2, 3]. A numerical solution of problems of this class by the finite-element method is given in [4]. One one-dimensional problem of forming was studied in [5].

In the present paper, we consider the problems of modeling of forming beams of a given curvature and angle of twisting in the regime of creep with allowance for elastic deformations. Strains and stresses are assumed to depend on two spatial coordinates and time only. Prior to forming, the profile is in a natural nondeformed state. The temperature is assumed to be constant during the entire time of the forming process.

1. Formulation of the Problem. In the general form, the direct problem is formulated as follows. During the time $0 \leqslant t < t_*$ (t is the time and t_* is a specified time of the forming process), the beam experiences specified external force and kinematic actions. It is assumed that the actions are such that the strains do not exceed the yield point. We have to determine the residual kinematic quantities with allowance for elastic recovery at the time $t = t_*$.

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Correspondingly, the inverse problem is formulated as follows. Which external actions have to be applied to the beam at $0 \leqslant t < t_*$ to obtain specified residual values after unloading at the time $t = t_*$ and elastic recovery?

From the practical point of view, inverse problems with three types of external actions are of interest.

Problem 1. At $t = 0$, as a result of instantaneous elastic deformation under the action of sought bending and torque and a specified longitudinal force, the beam acquires curvatures $æ_{x0}$ and $æ_{y0}$, angle of twisting θ_0, and longitudinal strain ε_0. At $0 < t < t_*$, the angle of twisting, curvatures, and longitudinal strain remain fixed; in this case, relaxation of stresses and accumulation of irreversible creep strains occur in the beam. At $t = t_*$, the beam is set free of all external loads and, after elastic recovery, has to acquire specified residual curvatures $æ_{x**}$ and $æ_{y**}$ and angle of twisting θ_{**}.

Problem 2. At $t = 0$, the beam instantaneously acquires specified curvatures $æ_{x0}$ and $æ_{y0}$, angle of twisting θ_0, and longitudinal strain ε_0; at $0 < t < t_*$, it additionally acquires sought constant rates $\dot{æ}_x$, $\dot{æ}_y$, $\dot{\theta}$, and $\dot{\varepsilon}_0$ (the dot indicates differentiation in time). At $t = t_*$, after unloading and elastic recovery, the beam has to acquire specified residual curvatures $æ_{x**}$ and $æ_{y**}$ and angle of twisting θ_{**}.

Problem 3. At $0 \leqslant t < t_*$, sought bending and torque and a specified longitudinal force are applied to the beam. At $t = t_*$, after unloading and elastic recovery, the beam has to acquire specified residual curvatures $æ_{x**}$ and $æ_{y**}$ and angle of twisting θ_{**}.

The following notation is used above: $\varepsilon_0(t)$ is the axial strain, $æ_x(t) = -v_{,zz}$ and $æ_y(t) = u_{,zz}$ are the curvatures of the projections $v(z,t)$ and $u(z,t)$ of the curved axis of the beam onto the planes yz and xz, respectively, in the Cartesian coordinate system xyz, and $\theta(t)$ is the angle of twisting.

The inverse problems 1 and 2 are solved using an iterative process similar to that described in [6]. The iterative algorithm of solving the inverse problem of pure twisting by a constant torque (problem 3) is based on the secant method.

The z axis of the Cartesian coordinate system is directed along the beam, and the x and y axes are in the cross-sectional plane. The assumption that the strains and stresses are independent of the z coordinate allows significant simplification of the system of the basic governing equations. Similar to [7, 8], the following assumptions are made for the beam:

$$\sigma_x = \sigma_y = \tau_{xy} = 0. \tag{1}$$

Then, elastic strains are related to stresses as follows:

$$\varepsilon_x^e = -\frac{\nu}{E}\,\sigma_z, \quad \varepsilon_y^e = -\frac{\nu}{E}\,\sigma_z, \quad \varepsilon_z^e = \frac{1}{E}\,\sigma_z, \quad \gamma_{zx}^e = \frac{1}{G}\,\tau_{zx}, \quad \gamma_{zy}^e = \frac{1}{G}\,\tau_{zy}.$$

Here E is Young's modulus, G is the shear modulus, and ν is the Poisson's ratio. In this case, the elastic strains

$$\varepsilon_z^e = \varepsilon_0 - æ_y x + æ_x y, \quad \varepsilon_x^e = -\nu\varepsilon_z^e, \quad \varepsilon_y^e = -\nu\varepsilon_z^e, \quad \gamma_{zx}^e = W_{,x} - \theta y, \quad \gamma_{zy}^e = W_{,y} + \theta x$$

are compatible and correspond to the displacements

$$u = -\nu(\varepsilon_0 x + æ_x xy + æ_y(y^2 - x^2)/2) + æ_y z^2/2 - \theta zy,$$

$$v = -\nu(\varepsilon_0 y - æ_y xy + æ_x(y^2 - x^2)/2) - æ_x z^2/2 + \theta zx,$$

$$w = \varepsilon_0 z + æ_x yz - æ_y xz + W.$$

Here $W(x,y,t)$ are the displacements along the z axis arising due to twisting.

If the creep of the material is described using the flow theory with allowance for damage accumulation in the material [9]

$$\eta_{kl}^c = \frac{3}{2(1-\omega)^m}\,\frac{F(\sigma_i)}{\sigma_i}\,\sigma_{kl}^0, \qquad \dot{\omega} = \frac{\Phi(\sigma_i)}{(1-\omega)^m}, \tag{2}$$

then, the total strain rates are determined by the relations

$$\dot{\varepsilon}_x = -\nu\dot{\sigma}_z/E - \eta_{zz}^c/2, \quad \dot{\varepsilon}_y = -\nu\dot{\sigma}_z/E - \eta_{zz}^c/2, \quad \dot{\varepsilon}_z = \dot{\sigma}_z/E + \eta_{zz}^c,$$

$$\dot{\gamma}_{zx} = \dot{\tau}_{zx}/G + \eta_{zx}^c, \quad \dot{\gamma}_{zy} = \dot{\tau}_{zy}/G + \eta_{zy}^c.$$

Here $\omega(x,y,t)$ is the damage of the material, η_{kl}^c are the strain rates of creep, $\sigma_i = (3/2)(\sigma_{kl}^0\sigma_{kl}^0)^{1/2}$ is the stress rate, and σ_{kl}^0 are the components of the stress deviator.

Relations (2) describe a material that is incompressible under creep. If condition (1) and the condition of elastic incompressibility of the material ($\nu = 1/2$) are satisfied, the total strain rates are also compatible, and the following relations are valid:

$$\dot{u} = -(1/2)(\dot{\varepsilon}_0 x + \dot{æ}_x xy + \dot{æ}_y (y^2 - x^2)/2) + \dot{æ}_y z^2/2 - \dot{\theta} zy,$$

$$\dot{v} = -(1/2)(\dot{\varepsilon}_0 y - \dot{æ}_y xy + \dot{æ}_x (y^2 - x^2)/2) - \dot{æ}_x z^2/2 + \dot{\theta} zx,$$

$$\dot{w} = \dot{\varepsilon}_0 z + \dot{æ}_x yz - \dot{æ}_y xz + \dot{W}.$$

In the general case of an elastically compressible material ($\nu \neq 1/2$), we have the following relations for the total strain rates $\dot{\varepsilon}_x$ and $\dot{\varepsilon}_y$:

$$\dot{\varepsilon}_x = \dot{\varepsilon}_y = -\dot{\varepsilon}_z/2 + (1/2 - \nu)\dot{\sigma}_z/E.$$

Then, the strain rates can be incompatible because of the second term, which takes into account the elastic compressibility of the material. It is assumed that the contribution of this term is insignificant in the case of developed creep strains and stationary external loads.

The equations for the total strain rates have the form

$$\dot{\tau}_{zx}/G + \eta^c_{zx} = \dot{W}_{,x} - \dot{\theta} y, \quad \dot{\tau}_{zy}/G + \eta^c_{zy} = \dot{W}_{,y} + \dot{\theta} x, \quad \dot{\sigma}_z/E + \eta^c_{zz} = \dot{\varepsilon}_0 + \dot{æ}_x y - \dot{æ}_y x. \tag{3}$$

With allowance for Eq. (1) and the fact that the stresses are independent of z, we have only one differential equation of equilibrium left out of three, and $\sigma_{z,z} = 0$. Then, we have

$$\tau_{zx,x} + \tau_{zy,y} = 0. \tag{4}$$

The stresses τ_{zx}, τ_{zy}, and σ_z should balance the force and external moments

$$N = \int\limits_S \sigma_z \, dS, \qquad M_x = \int\limits_S \sigma_z y \, dS, \qquad M_y = -\int\limits_S \sigma_z x \, dS; \tag{5}$$

$$M_z = \int\limits_S (\tau_{zy} x - \tau_{zx} y) \, dS, \tag{6}$$

where N, M_x, M_y, and M_z are the longitudinal force, bending, and torque. The boundary conditions on the cross-sectional contour of the beam are

$$\tau_{zx} n_1 + \tau_{zy} n_2 = 0 \tag{7}$$

(n_k are the components of the normal to the contour).

Equations (2)–(6) with the boundary conditions (7) form an integrodifferential system with respect to stresses, warping, and damage for Problems 1–3, and also the angle, curvatures, and tensile strain for Problem 3. At $t = 0$, the creep strains $\varepsilon^c_{ij}(x, y, 0) = 0$ and $\omega(x, y, 0) = 0$ and stresses

$$\tau_{zx}(x, y, 0) = G(W_{0,x} - \theta_0 y), \quad \tau_{zy}(x, y, 0) = G(W_{0,y} + \theta_0 x), \quad \sigma_z(x, y, 0) = E(\varepsilon^0_0 + æ_{0x} y - æ_{0y} x)$$

are determined from the equilibrium equation (4) for Problems 1–3 and relations (5), (6) for Problem 3 with allowance for the boundary conditions (7). Here $W_0 = W(x, y, 0)$, $\theta_0 = \theta(0)$, $æ_{0x} = æ_x(0)$, $æ_{0y} = æ_y(0)$, and $\varepsilon^0_0 = \varepsilon_0(0)$. For $t = t_*$, the conditions of elastic unloading are valid:

$$\int\limits_S (\tau_{zy*} x - \tau_{zx*} y) \, dS = \theta^e GD,$$

$$\int\limits_S \sigma_{z*} \, dS = \varepsilon^e_0 EJ, \qquad \int\limits_S \sigma_{z*} y \, dS = æ^e_x EJ_x, \qquad -\int\limits_S \sigma_{z*} x \, dS = æ^e_y EJ_y,$$

$$\tau_{zx*} = \tau^e_{zx} + \rho_{zx}, \qquad \tau_{zy*} = \tau^e_{zy} + \rho_{zy}, \qquad \sigma_{z*} = \sigma^e_z + \rho_z,$$

$$\theta_* = \theta^e + \theta_{**}, \qquad æ_{x*} = æ^e_x + æ_{x**}, \qquad æ_{y*} = æ^e_y + æ_{y**}, \qquad \varepsilon_{0*} = \varepsilon^e_0 + \varepsilon_{0**}$$

(ρ are the residual stresses after elastic unloading). The subscripts "$*$," "$**$," and e refer to the values at $t = t_*$ before unloading, residual values, and elastic recovery values; D, J, J_x, and J_y are the geometric characteristics of the section:

$$D = \int_S [(W^e_{,y} + x)x - (W^e_{,x} - y)y]\, dS, \quad J = \int_S dS, \quad J_x = \int_S y^2\, dS, \quad J_y = \int_S x^2\, dS.$$

Warping of the section $W^e(x, y)$ is proportional to the axial displacement of the cross-sectional point under elastic twisting $W = \theta W^e$ and is completely determined by the cross-sectional geometry of the profile.

To solve the system of differential equations (2)–(7), we used the finite-difference method. Equations that describe beam twisting usually reduce to one differential equation with respect to warping or the stress function with an appropriate transformation of the boundary conditions. Then, either the Dirichlet or the Neumann problem is solved at each time step. The values of warping or the stress function in grid nodes are used to determine the unknown stresses and strains for the next step. To avoid additional numerical differentiation of the right sides, we use the difference scheme

$$\frac{\dot\tau_{zx_{ij}}}{G} - \frac{\dot W_{i+1j} - \dot W_{i-1j}}{2h_1} = -\eta^c_{zx_{ij}} - \dot\theta y_{ij}, \qquad \frac{\dot\tau_{zy_{ij}}}{G} - \frac{\dot W_{ij+1} - \dot W_{ij-1}}{2h_2} = -\eta^c_{zy_{ij}} + \dot\theta x_{ij},$$

$$\frac{\dot\tau_{zxi+1j} - \dot\tau_{zxi-1j}}{2h_1} + \frac{\dot\tau_{zyij+1} - \dot\tau_{zxij-1}}{2h_2} = 0, \qquad \frac{\dot\sigma_{zij}}{E} = -\eta^c_{zij} + \dot\varepsilon_0 + \dot{æ}_x y_{ij} - \dot{æ}_y x_{ij}, \qquad \dot\omega_{ij} = \frac{\Phi_{ij}(\sigma_i)}{(1 - \omega_{ij})^m},$$

where $0 < i < K$, $0 < j < M$, $h_1 = a/K$, $h_2 = b/M$, a and b are the width and length of the rectangular cross section of the beam, and x_{ij} and y_{ij} are the coordinates of the node (i, j) of the finite-difference grid.

For the condition at the left boundary $\tau_{zx} = 0$ for $x = -a/2$, we use the approximation [10]

$$\frac{-\dot W_{2j} + 4\dot W_{1j} - 3\dot W_{0j}}{2h_1} = \dot\theta y_{0j}, \qquad 0 < j < \frac{b}{h_2}.$$

Equations for the remaining three boundaries of the section are written in a similar manner. The matrix of coefficients in the left side at derivatives in time for Problems 1 and 2 have a band structure. For Problem 3, Eq. (6) is written in the form

$$\int_S (\dot W_{,y} x - \dot W_{,x} y)\, dS + \frac{\dot\theta ab(a^2 + b^2)}{12} = \frac{\dot M_z}{G} + \int_S (\eta^c_{zy} x - \eta^c_{zx} y)\, dS.$$

The above-described schemes are used to approximate the derivatives with respect to x and y. Simpson's method [11] is used for numerical integration over the cross section. Thus, one more differential equation and, hence, one more row in the matrix of coefficients at derivatives in time are added. In this case, the matrix has no longer the band structure. Its conversion was performed by the method of reflection, which allows one to obtain the solution with a high accuracy and is stable to the computational error [10, pp. 265–268]. The method can be implemented both for the matrix of the general type and for the matrix with the band structure. To solve the system of ordinary differential equations in time, we used the fourth-order Runge–Kutta–Merson technique with an automatic choice of the step, which significantly reduced the time needed to solve the problem [11]. The number of simultaneously solved equations has the order $N = 5 \times K \times M$.

2. Numerical Solution. The proposed scheme was tested on a numerical solution of a linear elastic problem of twisting a beam with a rectangular section 0.01×0.02 m. Young's modulus $E = 66.7$ GPa and Poisson's ratio $\nu = 0.3$ correspond to the VT9 alloy.

Table 1 shows the values of the torque and maximum shear stress for $\theta = 1.396$ rad/m obtained by the finite-difference method with section splitting into 11×21 and 21×41 nodes, and also the values calculated by the formulas

$$M_z = k_1 G\theta a^3 b, \qquad \tau_{\max} = M_z/(k_2 a^2 b), \tag{8}$$

where the values of k_1 and k_2 depend on the ratio b/a [8] (here $k_1 = 0.229$ and $k_2 = 0.246$).

For elastic twisting, on the basis of the solution given in [7] for the stress function, we obtained a solution for stresses and warping in the form of a series, which is too cumbersome to be presented here. A comparison of stresses and warping obtained in the form of a series and by the finite-difference method indicates a rather high accuracy of the solution obtained by the finite-difference method.

TABLE 1

Calculation method	M_z, N · m	$\tau_{\max}$, MPa
Numerical solution:		
grid 11 × 21	163.3	331.0
grid 21 × 41	163.7	332.6
Solution (8)	164.0	332.8

TABLE 2

$\varepsilon_0 \cdot 10^3$	θ_0, rad/m	$æ_{x0}$, m^{-1}
0	1.184	0.52
1	1.183	0.51
2	1.172	0.46
3	1.143	0.38
4	1.082	0.30

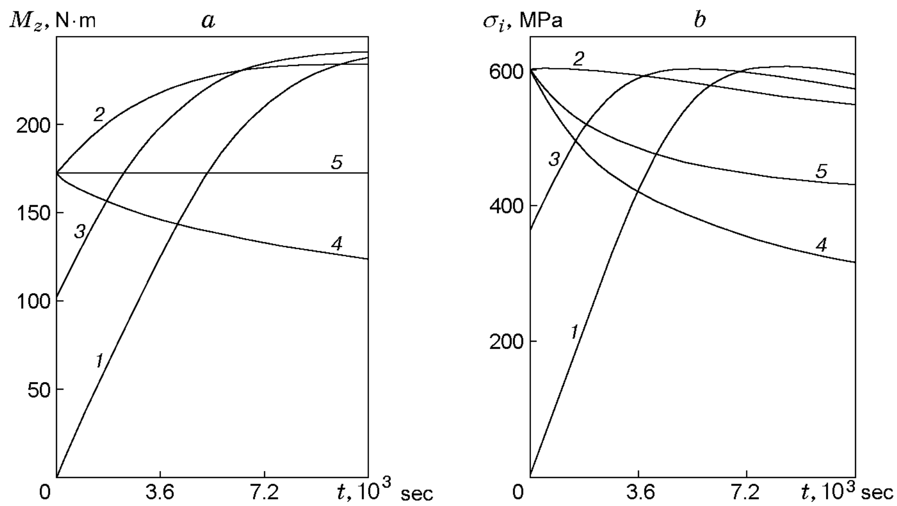

Fig. 1

Creep of the VT9 alloy is described by the relations

$$\eta^c_{kl} = B\sigma_i^{n-1}\sigma^0_{kl}/(1-\omega)^m, \qquad \dot{\omega} = C\sigma_i^g/(1-\omega)^m. \tag{9}$$

For a temperature of 550°C, the constants in relations (9) have the following values: $n = 4$, $g = 5$, $m = 10$, $B = 1.1303 \cdot 10^{-17}$ MPa$^{-n} \cdot$ sec^{-1}, $C = 5.0105 \cdot 10^{-20}$ MPa^{-g}, and the yield point is $\sigma_{\rm y} = 608$ MPa [13].

For Problem 1, Table 2 gives the sought initial angle of twisting θ_0 and curvature $æ_{x0}$ necessary for obtaining the residual values $\theta_{**} = 0.523$ rad/m and $æ_{x**} = 0.2$ m^{-1} for fixed tension ε_0 and $0 \leqslant t < t_*$. The time of heat fixing is $t_* = 3.6 \cdot 10^4$ sec.

In the calculations, the angle of twisting, curvature, and axial tension were chosen so that the maximum value of the stress rate for $0 \leqslant t < t_*$ did not exceed the yield point $\sigma_{\rm y}$.

Beam deformation with longitudinal tension fixed in time allows one to decrease the sought initial values of the angle of twisting and curvature, and hence, the values of elastic recovery at $t = t_*$.

Twisting and bending mutually affect each other. For instance, in the case of pure twisting, one has to set the angle of twisting $\theta_0 = 1.232$ rad/m to obtain $\theta_{**} = 0.262$ rad/m during the time $t_* = 3.6 \cdot 10^4$ sec; in the case of twisting with bending, to reach $\theta_{**} = 0.262$ rad/m and $æ_{x**} = 0.15$ m^{-1} during the same time, one needs the angle $\theta_0 = 1.05$ rad/m and $æ_{x0} = 0.62$ m^{-1}.

Figure 1 shows the numerical solution of direct problems of pure twisting for different regimes of deformation of the beam with the cross section 0.01 × 0.02 m for $t_* = 1.08 \cdot 10^4$ sec. Figure 1a shows the dependence of the torque M_z on time; the dependence of the maximum stress rate on time is plotted in Fig. 1b (the maximum stress rate is reached in the middle of the section boundary). Curves 1–5 refer to the following regimes: linear increase in the angle of twisting from 0 to 3.227 rad/m at $0 \leqslant t < t_*$ (curve 1), instantaneous elastic deformation at the initial time up to the angle of twisting of 1.465 rad/m with its subsequent linear increase to 4.239 rad/m at $0 < t < t_*$ (curve 2), instantaneous elastic deformation up to the angle of twisting of 0.872 rad/m with its subsequent linear increase up to 4.047 rad/m during the time t_* (curve 3), instantaneous elastic loading up to the angle of twisting of 1.465 rad/m with subsequent heat fixing (at $0 < t < t_*$, the angle of twisting remains unchanged, and stress relaxation occurs) (curve 4), and twisting by a constant moment at $0 \leqslant t < t_*$ (curve 5). The following residual

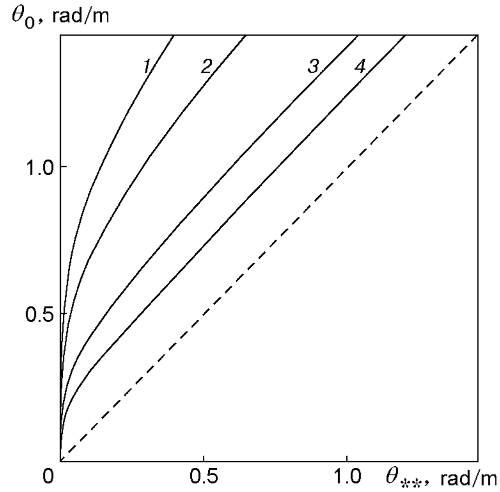

Fig. 2

TABLE 3

t_*, 10^3 sec	θ_*, rad/m	$æ_{x*}$, m^{-1}
7.2	2.936	1.35
10.8	2.742	1.25
18.0	2.529	1.14
36.0	2.283	1.02

TABLE 4

t_*, 10^3 sec	$\dot\theta$, 10^{-4} rad/(m·sec)	θ_{**-}, rad/m	θ_{**+}, rad/m	$\omega_{\max}$
3.6	1.744	0.363	0.368	0.01
36.0	1.744	5.757	5.943	0.09
36.0	2.617	8.691	9.022	0.13
72.0	1.744	12.037	12.401	0.15

angles were obtained for deformation regimes corresponding to curves 1–5: $\theta_{**} = 1.195$, 2.243, 1.987, 0.412, and 0.801 rad/m, respectively.

Figure 2 shows the dependence of the initial angle of twisting on the given residual angle (Problem 1). Curves 1–4 refer to the times of heat fixing $t_* = 1.08 \cdot 10^4$, $3.6 \cdot 10^4$, $36 \cdot 10^4$, and $180 \cdot 10^4$ sec, respectively. The dashed curve is the asymptotic solution for $t_* \to \infty$.

The results of the solution of Problem 2 are listed in Table 3. At $t = 0$, the beam is instantaneously loaded up to the angle of twisting $\theta_0 = 1.047$ rad/m and curvature $æ_{x0} = 0.5$ m^{-1}, which increase linearly at $0 < t < t_*$ up to the sought angle θ_* and curvature $æ_{x*}$ necessary to obtain the residual angle $\theta_{**} = 1.221$ rad/m and curvature $æ_{x**} = 0.5$ m^{-1}.

We also studied the influence of damage on calculation results. The data for pure twisting (without bending and tension) are given below. At $t = 0$, the beam is instantaneously loaded up to the angle of twisting $\theta_0 = 1.396$ rad/m; at $0 < t < t_*$, the angle changes with a given velocity $\dot\theta$. Table 4 shows the residual angles after unloading for different values of t_* and $\dot\theta$ without allowance for damage (θ_{**-}) [$m = 0$ in (9)] and with allowance for damage (θ_{**+}), and also the maximum value of the damage $\omega_{\max}$ if it was taken into account.

It follows from the analysis of the data obtained that the damage should be taken into account in Problem 2 if the forming process is rather long.

3. Experimental Data. *Twisting of Square-Section Beams.* Experimental data obtained in pure twisting by a constant torque (Problem 3) are compared with the results of the numerical solution for beams with a square cross section and close to it. The square form of the cross section allows one to maximally reduce the influence of warping constraint at the butt-end faces of the specimen on test results.

Figure 3 shows the specimen with cross-sectional dimensions 0.01×0.02 m and working length l_0 of about 0.07 m. It is seen that longitudinal twisting of the specimen is not uniform: it is stronger in the central part and weaker at the ends. This character of twisting is confirmed by direct measurements by an instrumental microscope.

The open squares 1 and 2 in Fig. 4 refer to the experimental dependence of the angle of twisting θ on time for beams of square cross section made of the St. 45 construction material (from a bar 120 mm in diameter) at temperatures $T = 725$°C ($l_0 = 0.051$ m, $a = b = 0.018\,15$ m, and $M_z = 49.57$ N·m) and $T = 740$°C ($l_0 = 0.0497$ m, $a = 0.018\,13$ m, $b = 0.018\,61$ m, and $M_z = 43$ N·m), respectively. The solid curves 1 and 2 correspond to the numerical calculations by the finite-difference method for $T = 725$ and 740°C. For calculations at the temperature

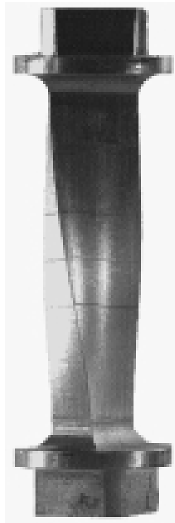

Fig. 3

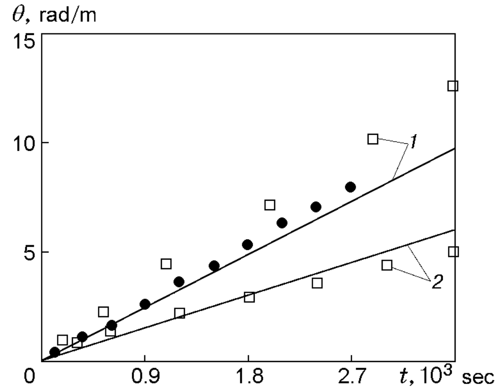

Fig. 4

of 725°C, we used the power dependence for the creep strain rates

$$\eta_{kl}^c = B\sigma_i^{n-1}\sigma_{kl}^0 \tag{10}$$

with the following constants of the material [14]: $n = 5.22$, $B = 3.5 \cdot 10^{-14}$ MPa$^{-n} \cdot$sec^{-1}, $E = 170$ GPa, and $\nu = 0.3$. The constant for calculations at the temperature of 740°C are given below.

Kachanov [15] gives the solution of the problem of twisting of a square-section beam under the assumption of steady creep (the changes in elastic strain rates are neglected) for the power dependence of the strain rate of creep, which was obtained by an approximate analytical method based on the principle of the minimum additional scattering:

$$\dot{\theta} = (3^{3(n+3)/2}B/a)(M/a^3)^n I(n). \tag{11}$$

Here $I(n)$ is the integral calculated by the Gaussian technique [in our case, $I(5.22) \approx 0.809$]. The difference in velocities calculated by formula (11) [$\dot{\theta} = 2.73 \cdot 10^{-3}$ rad/(m · sec)] and by the finite-difference method [$\dot{\theta} = 2.66 \cdot 10^{-3}$ rad/(m · sec)] is less than 3%.

The filled points in Fig. 4 refer to the experimental values of the angle of twisting of the continuous circular-section beam in the case of its twisting by the torque $M_z = 51.5$ N · m at $T = 725$°C (the beam radius is $R = 9.988 \cdot 10^{-3}$ m; $l_0 = 0.0431$ m). The velocity of the angle of twisting $\dot{\theta} = 2.73 \cdot 10^{-3}$ rad/(m · sec) calculated for this beam under the assumption of steady creep [15],

$$\dot{\theta} = (\sqrt{3}B/R^{3n+1})(M\sqrt{3}(3 + 1/n)/(2\pi))^n, \tag{12}$$

coincides with that calculated by Eq. (11) for a square-section beam. The experimental values of $\theta(t)$ for beams with square (open squares 1 in Fig. 4) and circular (filled points in Fig. 4) cross sections differ insignificantly. It should be noted that good agreement of experimental data for twisting of square-section beams and experimental data for twisting of circular-section beams is important from the viewpoint of planning the experiment (torque, specimen size, test duration, etc.).

An analysis of formulas (11) and (12) shows that the velocity of the angle of twisting depends significantly on the cross-sectional size: $\dot{\theta} \sim 1/a^{3n+1}$. A 1% decrease in a for $n = 5.22$ leads to an increase in $\dot{\theta}$ approximately by 20%.

For the calculations for $T = 740$°C (solid curve 2 in Fig. 4), we used the dependence

$$\eta_{kl} = A(\exp(\alpha\sigma_i) - 1)\sigma_{kl}^0/\sigma_i, \tag{13}$$

where $\alpha = 0.125\,63$ MPa^{-1} and $A = 8.0451 \cdot 10^{-8}$ sec^{-1}. Young's modulus and Poisson's ratio are the same as those at the temperature $T = 725$°C. The velocity of the angle of twisting $\dot{\theta} = 1.67 \cdot 10^{-3}$ rad/(m · sec) obtained by the finite-difference method is smaller than the experimental value approximately by 25%.

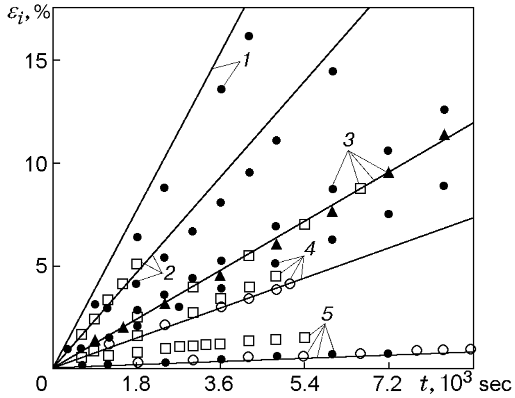

Fig. 5

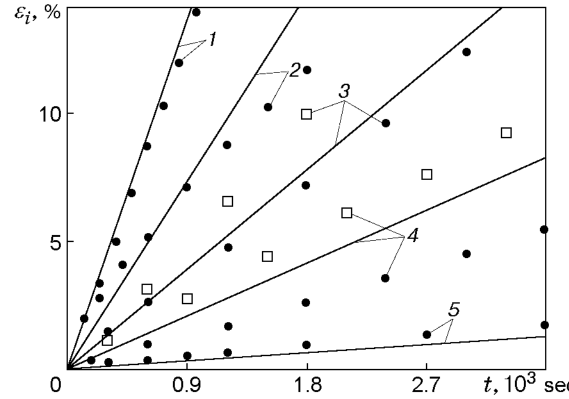

Fig. 6

With allowance for the scatter of experimental data obtained in determining the characteristics of creep and the strong dependence of the calculation results on the cross-sectional size of the specimen, the agreement of experimental and numerical data may be considered as satisfactory.

Determination of Characteristics of Creep. Lyubashevskaya and Sosnin [14] calculated the constants for the power law of creep (10) on the basis of experimental data of scattered energy versus time in experiments on tension and compression for the St. 45 material for $T = 725$°C. It was assumed that the material behaves as an isotropic medium without hardening, with identical properties in tension and compression.

Figure 5 shows the test results for pure tension and compression in the axial direction (filled and open points, respectively) and tension at an angle of 45° to the bar axis (triangles) at constant stress and $T = 725$°C. These data confirm the isotropy of the material. The experimental curves 1–5 correspond to stresses $\sigma = 55$, 50, 44, 40, and 26 MPa. The squares refer to the experimental values of the strain rate at the characteristic point $\hat{\varepsilon}_i$ in twisting of continuous circular-section specimens under the action of a constant torque M_z [16]. The experimental data obtained in twisting correspond to the stress rate at the characteristic point, equal to stress in pure tension: $\hat{\sigma}_i = \sigma = 26$, 40, 44, and 50 MPa ($M_z = 31.2$, 42.8, 53.1, and 60.3 N$\cdot$ m and $R = 9.975 \cdot 10^{-3}$, $9.988 \cdot 10^{-3}$, 0.01, and $9.995 \cdot 10^{-3}$ m). The length of the specimens is 0.047 m. The values at the characteristic point are marked by hat sign.

The position of the characteristic point is determined as the coordinate of intersection of the curves corresponding to elastic and steady distributions of stresses [16]. As applied to twisting of continuous circular-section beams by a constant torque, when the creep index n is unknown, it is possible to use the coordinate of intersection of the curves corresponding to elastic and ideally plastic distributions ($\hat{R} = 3R/4$); the accuracy is sufficient to construct twisting diagrams. In this case, we have $\hat{\varepsilon}_i = 3R\theta/4$ and $\hat{\sigma}_i = \sqrt{3}\, 3M_z/(2\pi R^3)$.

Figure 6 shows the experimental dependences (points) of the strain rate in pure tension in the axial direction on time for $T = 740$°C. The experimental values numbered 1–5 correspond to $\sigma = 60$, 55, 50, 45, and 30 MPa. Squares 3 and 4 in Fig. 6 refer to the strain rates at the characteristic point, which were obtained in the experiment on twisting a continuous circular-section specimen with the stress rate at the characteristic point $\hat{\sigma}_i = 50$ and 45 MPa ($M_z = 60.4$ and 54.4 N$\cdot$m and $R = 9.996 \cdot 10^{-3}$ m).

The solid curves in Figs. 5 and 6 approximate the experimental data by the power dependence (10) for $T = 725$°C and the exponential dependence (13) for $T = 740$°C, respectively. It follows from the data in Figs. 5 and 6 that the experimental points are grouped along a straight line for an identical stress rate $\sigma_i = \hat{\sigma}_i$, which confirms the hypothesis of the "single curve" and the choice of the equivalent stress.

Thus, the proposed technique for modeling and calculating the parameters of beam forming under conditions of creep on the basis of the finite-difference method finds satisfactory confirmation for the problem of determining the torque necessary to reach a required angle of twisting for a beam whose cross section is close to a square.

The finite-difference calculation allows one to analyze the sensitivity of strain rates to changes in the geometric dimensions of the specimen, which is important for planning and performing of experiments.

A further experimental study of twisting of beams of more complicated cross-sectional shapes (including the case of large strains) is necessary to test the calculation method for beam forming.

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